RYERSON UNIVERSITY

DEPARTMENT OF MATHEMATICS

MTH 210 MIDTERM - WINTER 2005

NAME: ______________________________

STUDENT ID: ______________________________

SECTION: ______________________________

Section	**Lab**	**TA**	**Section**	**Lab**	**TA**
1	Thursdays at 1	Anca	2	Thursdays at 4	Chris
3	Thursdays at 5	Chris	4	Fridays at 9	Anca

INSTRUCTIONS

This exam has 5 pages including this front page. It consists of 3 questions and is worth 25% of the course mark. Please answer all questions directly on this exam.

This is a closed book exam. One 8.5" by 11" double-sided crib sheet is allowed, but no other aids are.

This exam is 2 hours long.

If you need more room for the solutions, please continue on the back of the page and indicate CLEARLY that you have done so.

Question A – Recursion and Induction	20
Question B – Number Theory	20
Question C – Graph Theory	20

Question A – Recursion and Induction – 20 Marks

Given the sequence a_n defined with the recurrence relation:

$a_0 = 1$

$a_n = k (a_{n-1})^2$ for $n \geq 1$

Terms of a Sequence (5 marks)

Calculate a_1, a_2, a_3, a_4, a_5

Keep your intermediate answers as you will need them in the next question.

Iteration (3 marks)

Using iteration, solve the recurrence relation when $n \geq 1$ (i.e. find an explicit formula for a_n).

Simplify your answer as much as possible. Your final solution can contain the symbols $\prod$ and $\sum$.

Proof by induction (12 marks)

Show that $2 \mid n^2 - n$ for all positive integers n by weak induction. No other method is acceptable.

Question B – Number Theory – 20 marks

Euclidian Algorithm (5 marks)

Use the Euclidian Algorithm to find gcd (598, 1287). Show all your steps.

Mod Proof (15 marks)

Prove that for any integers A, B, a, d such that $d \neq 0$,
if A mod d = a and B mod d = 1 then AB mod d = a

Question C – Graph Theory – 20 marks

Graph Degrees (12 marks)

For each of the following questions, either draw a graph with the requested properties, or explain **convincingly** (possibly by quoting a theorem) why such a graph cannot be drawn.

a) A graph with 5 vertices of degrees 5, 5, 4, 4, 3

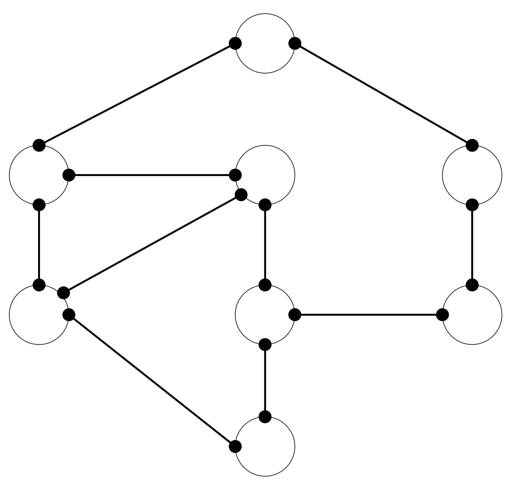
b) A graph with 5 vertices of degrees 5, 5, 4, 4, 4

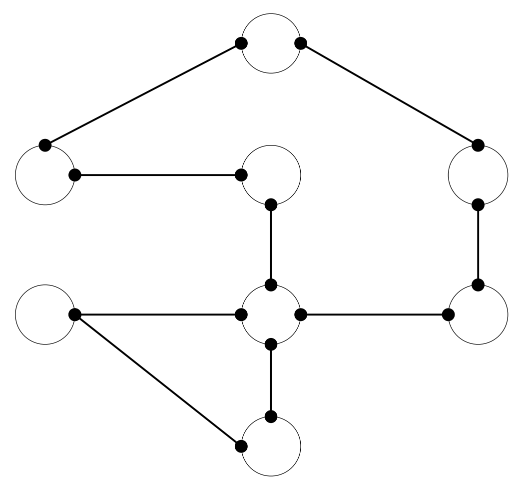
c) A simple graph with 5 vertices of degrees 5, 5, 4, 4, 4

d) A simple graph with at least 5 vertices which have degrees 5, 5, 4, 4, 4. The other vertices have whichever degree seem appropriate.

<u>Circuits (8 marks)</u>

Find the requested circuits in the following graphs or explain why they don't exist, supporting your explanation with a theorem. Give the circuits that you can find as a sequence of vertices and edges, for example: $v_1e_2v_2e_3v_1$





a) Find an Euler circuit in G that starts at v_1

c) Find an Euler circuit in H that starts at v_1

b) Find a Hamiltonian circuit in G that starts at v_1

d) Find a Hamiltonian circuit in H that starts at v_1

v1

e1

e3

v2

v3

e4

e6

e5

v5

v6